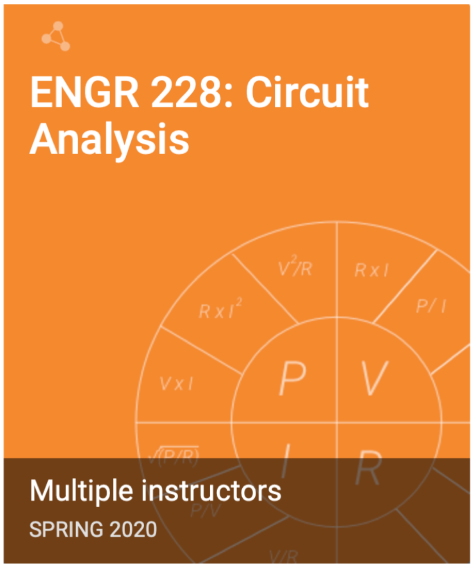


Chapter 5.2

First Order Circuits

Capacitance

Engr228 - Circuit Analysis

Spring 2020

Dr Curtis Nelson

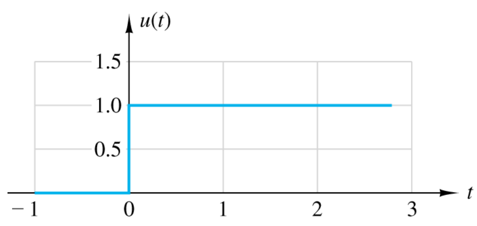
Section 5.2 Objectives

- Learn to:
  - Define the electrical properties of a capacitor, including its i-υ relationship and energy equation.
  - Combine multiple capacitors when connected in series or in parallel.

The Step Function u(t)

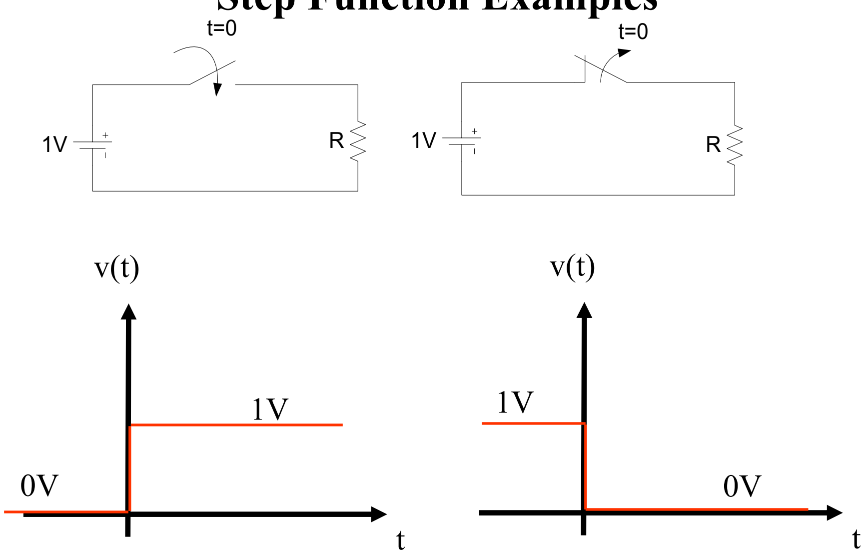
First, we will define a **Step Function**:

- A *Step Function* is often used to drive circuits.
- The Step Function 1*u(t)* represents a function that has zero value up until t = 0, and a value of 1 forever after.



(a)

Step Function Examples



Unit Step function

Capacitance

- The capacitor is an energy storing device;
- The ***Capacitance*** (C) is a measure of the capacitor's potential to store energy in an **electric** field;
- A capacitor is constructed of two conducting plates separated by an insulator.

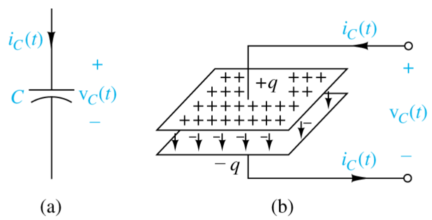
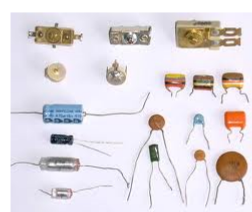


Figure 7.15 (a) Symbol for the capacitor with conventional voltage and current directions. (b) Illustration of electric field between plates of a parallel-plate capacitor.

Real Capacitors

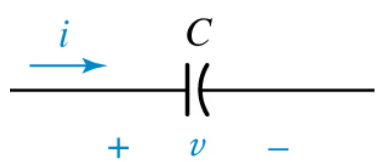


Practical Definition of Capacitance

- **Capacitance** is the ability of a body to hold an electrical charge. A common form of an energy storage device is the parallel-plate capacitor where the capacitance is directly proportional to the surface area of the conductor plates and inversely proportional to the separation distance between the plates. If the charges on the plates are $+Q$ and $-Q$, and V gives the voltage between the plates, then the capacitance is given by:

$$C = Q/V$$

Capacitors



- The unit of capacitance is the Farad (F);
- 1 F = 1 Amp-Second/Volt = 1 Coulomb/Volt;
- The governing voltage and current relationship is:

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

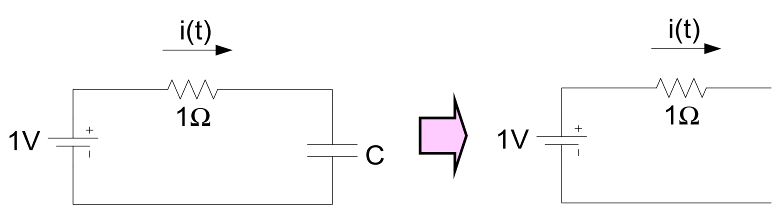
More on Capacitance

- The capacitance used in electronic circuits is typically several orders of magnitude smaller than the farad. The most common units of capacitance in use today are the micro-farad (μF), nano-farad (nF), and pico-farad (pF).
- The capacitance of a *parallel-plate* capacitor constructed of two parallel plates, both of area *S* separated by a distance *d,* is approximately equal to the following:

$$C = \varepsilon_r \varepsilon_0(S/d)$$

  - *C* is the capacitance;
  - ε_r is the relative static permittivity (sometimes called the dielectric constant) of the material between the plates (for a vacuum, $\varepsilon_r = 1$);
  - ε_0 is the electric constant ($\varepsilon_0 \approx 8.854 \times 10^{-12}$ Fm^{-1});
  - *S* is the area of overlap of the two plates;
  - *d* is the separation between the plates.

DC Characteristics of a Capacitor

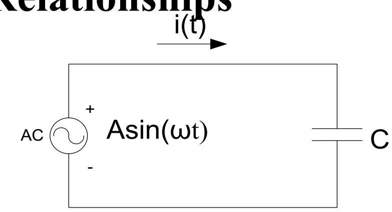


The capacitor acts like an "open circuit" at DC because the time rate of change of voltage is zero so, no current can flow through it.

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

Capacitor IV Relationships

From the circuit at the right, find i(t).



$$i(t) = C\frac{dv(t)}{dt} = C\frac{d(A\sin\omega t)}{dt}$$

$$= A\omega C(\cos\omega t)$$

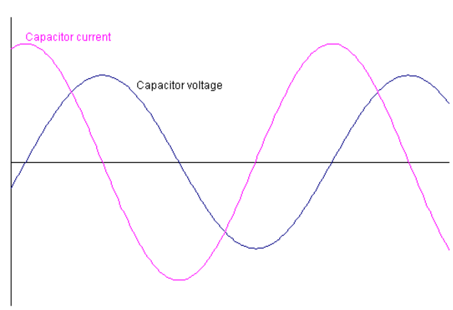
$$= \frac{A}{\left(\frac{1}{\omega C}\right)}\sin(\omega t + \frac{\pi}{2})$$

Phase shift of +90º

The impedance of the capacitor is called ***Capacitive Reactance***.

Voltage-Current Relationship in a Capacitor

Current and voltage in a capacitor are not in phase with each other. **For sinusoidal waves, the voltage across a capacitor lags the current through it by 90º.** (In other words, the current leads the voltage by 90º.) In the diagram below, the tall purple waveform represents the current through a capacitor and the shorter blue waveform represents the voltage across a capacitor.



Power and Energy in a Capacitor

Power in a capacitor:

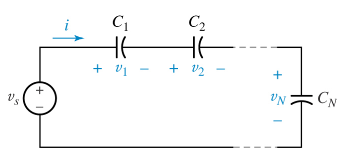
$$p_C = v_C i_C = C v_C \frac{dv_C}{dt}$$

Energy is stored in the ***Electric*** field in a capacitor:

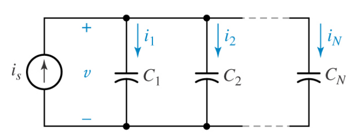
$$w_C(t) = \int_{t_0}^{t} p\,dt$$

$$w_C(t) = \frac{1}{2} C {v_C}^2$$

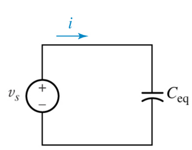
Capacitor Combinations



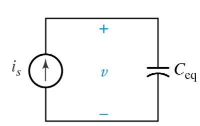
(a)



(c)



(b)



(d)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

$$C_{eq} = C_1 + C_2 + \ldots + C_N$$

Section 5.2 Summary

- You learned to:
  - Define the electrical properties of a capacitor, including its i-υ relationship and energy equation.
  - Combine multiple capacitors when connected in series or in parallel.